

8.5 Nuclear radius

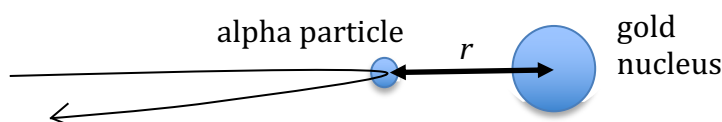
We can use two techniques to find the radius of an atomic nucleus.



videos

Closest approach method

In the Rutherford scattering experiment, alpha particles are fired at a thin gold foil. Some of the alpha particles are detected coming straight back from the gold foil. This indicates that the positively charged alpha particles are being repelled by the positively charged gold nucleus. At the point of closest approach, the initial kinetic energy of the alpha particle E_k (which is a fixed value for a particular radioactive source) is totally transferred to electrical potential energy E_p .



We can write:

$$E_k = E_p$$

$$\therefore \frac{1}{2}m_\alpha v^2 = \frac{1}{4\pi\epsilon_0} \frac{Q_\alpha \times Q_g}{r}$$

where m_α = mass of alpha particle, Q_α = charge of alpha particle, Q_g = charge of gold nucleus, r = distance of closest approach.

(1) Rearrange the equation to make r the subject.

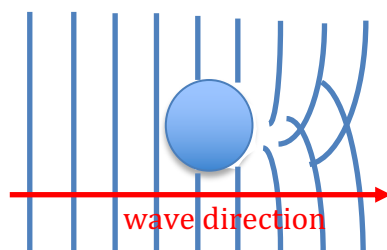
(2) Taking $m_\alpha = 6.64 \times 10^{-27} \text{ kg}$, $v = 2 \times 10^7 \text{ ms}^{-1}$, $Q_\alpha = 2 \times 1.6 \times 10^{-19} \text{ C}$, $Q_g = 79 \times 1.6 \times 10^{-19} \text{ C}$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$, work out a value for r .

(3) How does the value of r calculated compare to the size of the nucleus?

(4) The closest approach method produces an upper limit on the size of the nucleus. Why is this?

Electron diffraction

Electrons accelerated to close to the speed of light (c) have wave like properties. They can be used to investigate the size of the atomic nucleus.



Wave fronts passing an object and diffracting.


The de Broglie wavelength can be calculated:

$$\lambda = \frac{h}{p}$$

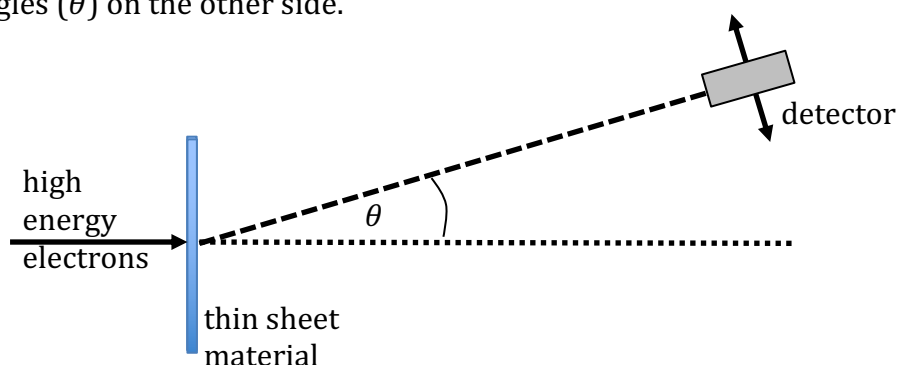
where h = the Planck constant = 6.63×10^{-34} Js, p = momentum of electron.

[As the electron is travelling close to the speed of light, relativistic calculations need to be used to work out the de Broglie wavelength. This is beyond the scope of this course.]

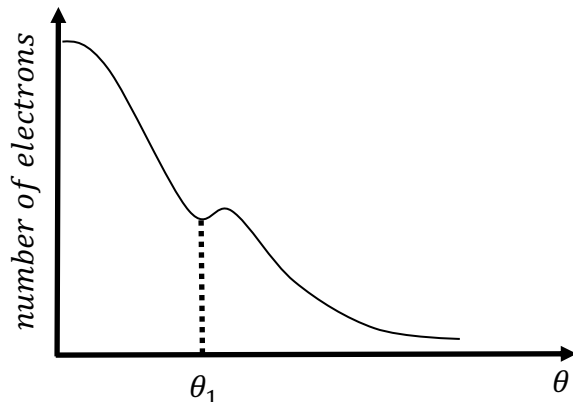
Electrons need to be accelerated in an electric field across several hundred megavolts.

(5)  An electron is accelerated through 400 MV. What energy does the electron gain?

These high energy electrons are passed through a thin sheet of a solid material (in a vacuum) and a detector is used record the arrival of electrons at different angles (θ) on the other side.



A graph of the number of electrons versus angle is shown below:




There are two effects taking place.

- 1) Electrons are being scattered. This means that they are being deflected when they pass close to the nucleus. This is similar to alpha particle scattering, except that, in this case the electrons are attracted to the nucleus. Scattering produces the overall decrease in the number of electrons detected as the angle increases.
- 2) Electrons are being diffracted and producing an interference pattern. This is responsible for the small dip at an angle θ_1 . This is a first order minima.

The angle at which the dip is detected can be used to calculate the radius r of the nucleus.

$$r \sin \theta_1 = 0.61\lambda$$

where $\lambda =$ de Broglie wavelength of the electrons.


(6)  electrons with a de Broglie wavelength $\lambda = 3 \times 10^{-15} \text{ m}$ are fired at a target containing oxygen nuclei. A first order minima is detected at an angle of 44° . Estimate the radius on the oxygen nucleus.


Nuclear radii for different elements

Electron diffraction has been used to find the radii of the nuclei of different elements with atomic mass number (or 'nucleon number') A . It is found that there is a simple relationship between atomic mass number A and radius r .

$$r = r_0 A^{\frac{1}{3}}$$

where $r_0 = 1.05 \times 10^{-15} \text{ m}$.


(7)  What is the radius of a hydrogen nucleus?


(8)  What is the radius of a nitrogen nucleus?


Nuclear density

The nuclear density can be calculated:

$$\rho = \frac{\text{nuclear mass}}{\text{nuclear volume}} = \frac{M}{\frac{4}{3}\pi r^3}$$

(9)  Calculate the nuclear density for hydrogen.

(10)  Substitute $r = r_0 A^{\frac{1}{3}}$ in the equation for density, above, and simplify.

(11)  How does the nuclear density vary (if at all) with the atomic mass number A ? Explain.