

## 7.3.1 Gravitational potential energy

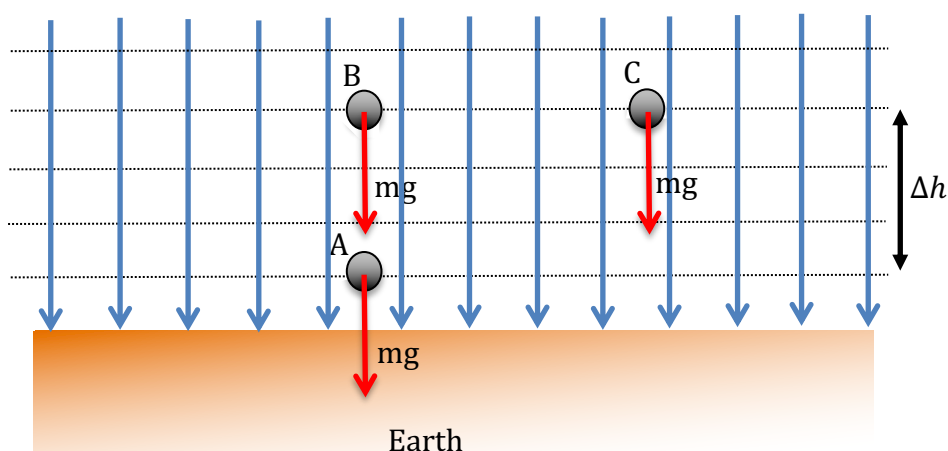
### Uniform fields

We know that when we lift a mass ( $m$ ) to a height ( $\Delta h$ ) above the Earth's surface the mass gains gravitational potential energy:

$$\Delta E_{gp} = mg\Delta h$$



This is strictly only an approximation when the height is small compared to the distance to the centre of the Earth. In this case, we assume that the gravitational field is uniform and  $g$  doesn't change with height.



In the diagram, above, we can see that if we move a mass ( $m$ ) from position A to position B, work is done against the force of gravity ( $=mg$ ).

$$W = \text{force} \times \text{distance moved in the direction of the force}$$

$$W = mg\Delta h$$

(1) *The force is only in the vertical direction. What work would be done in moving the mass from A to C?*

(2) *What work is done in moving the mass from B to C?*

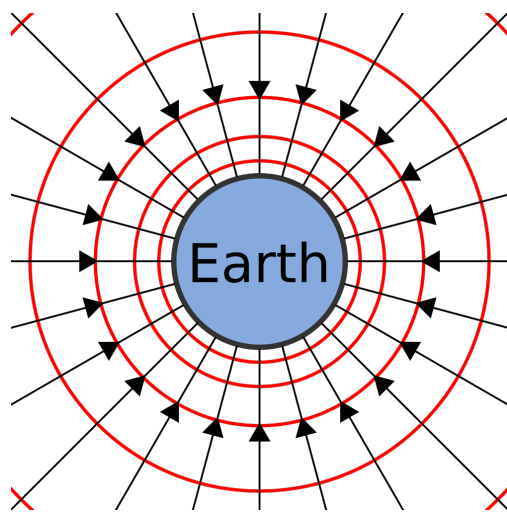
The work done in moving the mass is equal to the change in gravitational potential energy.

(3) *Is the change in potential energy positive or negative in when moving a mass from A to B?*

The dotted lines in the diagram are called equipotential lines. No work is done in moving a mass along an equipotential line, and there is no change in gravitational potential energy.

### Radial fields

Let us consider the gravitational field produced by a spherical mass, such as the Earth:



On a bigger scale, we can see that the gravitational field around the Earth is not uniform.

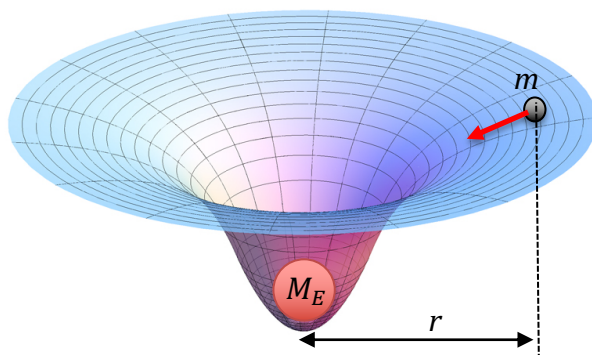
(4) *How can you tell from the field lines that the field strength decreases as you move away from the Earth?*

(5) *What are the circular lines?*

The gravitational potential energy at a certain point in this field is defined as:  
*"the work done in moving a mass from infinity that point"*

A good way to visualise gravitational potential energy is to think about a stretched membrane. A mass (such as the Earth) will cause a dip in the membrane. Another mass will tend to roll down into this dip. It will lose gravitational potential energy as it rolls down the slope. See also:

<http://www.falstad.com/vector/>




At a separation of infinity, the mass is defined to have zero gravitational potential energy.


This means that the gravitational potential energy of the mass ( $m$ ) is always negative, because it is losing gravitational potential energy as it moves towards  $M$ .

The gravitational potential energy for a mass  $m$  in a gravitational field caused by mass  $M$  is given by the expression:


$$E_{gp} = -\frac{GMm}{r}$$

where  $G$ =the gravitational constant= $6.67 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$ , and  $r$ =distance between the centre of mass of  $M$  and  $m$

(6)  What is the gravitational potential energy of a person with a mass of 80kg on the surface of the Earth? (mass of Earth =  $6 \times 10^{24}$ kg, radius of Earth = 6400km)

(7)  If the person was launched into space to twice the radius of the Earth, what would their gain in gravitational potential energy ( $\Delta E_{gp}$ ) be?

The next question is tricky!

(8)  The escape velocity of the Earth is defined to be the launch speed required to overcome the gravitational attraction of the Earth. From conservation of energy considerations, work out the escape velocity of the Earth.