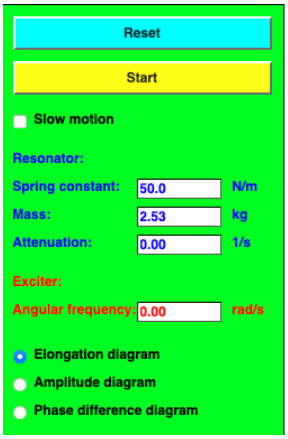
## 6.4 Forced vibrations and resonance

Forced vibrations occur when two systems are coupled together, and you have a DRIVER and a RESPONDER. The driver (or exciter) provides a periodic force to the responder (or resonator).

videos

Open the following simulation:

https://www.walter-fendt.de/html5/phen/resonance_en.htm




Make sure that the resonator has the settings shown in the diagram to the right.

The resonator has a natural frequency at which it likes to oscillate (You might like to look back at section 6.3.1.). The period of oscillation for a spring pendulum is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

*(1) Use the values given (right) to work out the natural frequency ($f = \frac{1}{T}$) and the natural angular frequency ($\omega = 2\pi f$) for this spring pendulum. Show your working.*

*(You should find that the natural angular frequency $\omega =$ 4.45 $rad/s$.)*

The angular frequency of the exciter (driver) can be changed in this simulation.

*(2) Without changing the spring constant or mass, change the exciter (driver) angular frequency (in red) and record the amplitude of the resonator (i.e. A). Record your values in the table below:*

| *angular frequency of driver ($rad/s$)* | *responder amplitude ($cm$)* |
|---|---|
| *3.00* | |
| *3.50* | |
| *4.00* | |
| *4.50* | |
| *5.00* | |
| *5.50* | |

*(3)✎ What conclusions do you draw from your results?*

The maximum amplitude for the oscillating responder occurs when the frequency of the driver matches the natural frequency of the responder. At this point the system is said to be resonating, and the driver is feeding in energy to the system.

*(4)✎ Set the driver (exciter) angular frequency to* $4.45\,rad/s$ *(i.e. at the resonant frequency). What do you notice?*

Without damping (attenuation), energy in the system would continue to build up until the system breaks! Damping removes energy from the system.

**Phase difference between driver and responder**

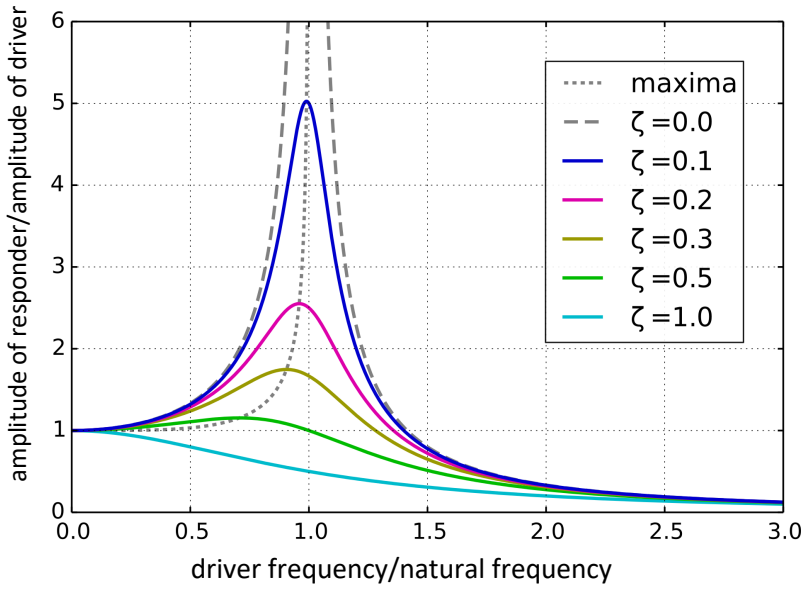
With the attenuation set to 0.2, set the angular frequency to the following values in turn: $3.50\,rad/s$, $4.50\,rad/s$, $5.50\,rad/s$.

Carefully observe the phase difference between driver and responder (make sure that Elongation diagram is selected).

You should see that the driver and the responder are nearly in phase (zero phase difference) for small angular frequencies (e.g. $3.50\,rad/s$). At resonance (around $4.50\,rad/s$, the driver is one quarter of a cycle $\left(\frac{\pi}{2}\ rad\ or\ 90°\right)$ ahead of the responder mass.

*(5)✎ What do you notice about the phase difference between the driver and responder for higher angular frequencies (e.g.* $5.50\,rad/s$*)?*

## Amplitude – frequency graph




The graph, left, shows how the amplitude of the responder changes for different driver frequencies. Lines are drawn for different degrees of damping (attenuation). The greater the damping, the smaller the amplitude. The dotted line shows the resonant frequency. Note that resonant frequency occurs when the driver frequency is approximately equal to the natural frequency when damping is small but moves to a lower frequency as the damping increases.

## Barton's pendulums

*(6)✎ Do some internet research on Barton's pendulums. Sketch what they look like and describe what occurs when the driver pendulum starts oscillating.*

*(7)✎ Explain why this is observed. You should refer to the equation for the period of a pendulum.*