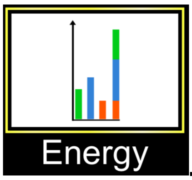
## 6.3.2 Energy and simple harmonic motion

videos

A **freely oscillating system** will not experience any loss of energy (e.g. through friction) and so will continue oscillating for ever.

Consider a simple pendulum. Run the following simulation:

https://tinyurl.com/y86t8rcl

And select the 

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*(1)✏Describe the energy changes as the pendulum oscillates.*

The pendulum moves with a displacement given by:

$$x = A\cos\omega t \quad \text{(where } \omega = 2\pi f \text{ and } A \text{ is the amplitude)}$$

Therefore, the velocity of the pendulum is given by:

$$v = -A\omega\sin\omega t$$

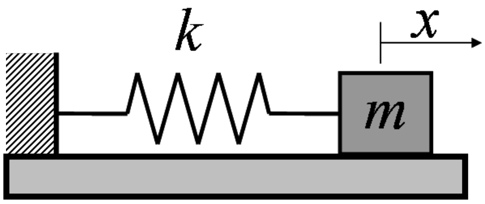
The maximum kinetic energy occurs where the pendulum is moving fastest. The maximum speed is given by:

$$v_{max} = A\omega \quad \text{(i.e. where the sin function is a maximum = 1)}$$

*(2)✏Write an expression for the maximum kinetic energy (k.e.$_{max}$) of the pendulum.*

*(3)✏If we take it that all the gravitational potential energy has transferred to kinetic energy at the bottom of the swing, write and expression for the maximum gravitational potential energy (g.p.e.$_{max}$).*

Now consider a mass-spring system where the mass oscillates backwards and forwards on a frictionless surface




We know that the elastic energy ($E_{elas}$) stored in a spring is given by:

$E_{elas} = \frac{1}{2}kx^2$ *(where k is the spring constant and x is the displacement from the equilibrium position)*

This reaches a maximum when the spring is at its maximum displacement from equilibrium (i.e. where $x = A$). At this point the total energy $E_{tot}$ of the oscillating system is stored in the spring as elastic energy.

The kinetic energy is therefore given by:

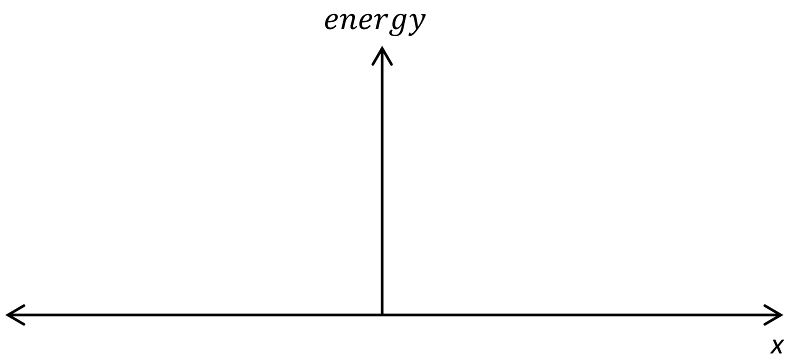
$$E_k = E_{tot} - E_{elas} = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2) \qquad (1)$$

Given that the kinetic energy $E_k$ is given by:

$$E_k = \frac{1}{2}mv^2 \qquad (2)$$

*(4)✏Equate the two equations, above, and rearrange to find an expression for the velocity v at a particular displacement x (remember that k and A are constants).*

*(5)✏Sketch a graph to show how $E_k$ and $E_{elas}$ (which is the elastic potential energy) change with displacement x.*




**Damped oscillations**

Free oscillations don't involve any loss in energy, so that the energy remains in the oscillating system. There is a continuous transfer in energy from potential to kinetic energy.

In damped systems, energy is removed from the system as time increases. The amount of damping reflects how quickly energy is removed.

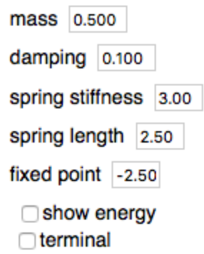
*Light damping (energy removed slowly)*

Click on the following link:

https://www.myphysicslab.com/springs/single-spring-en.html

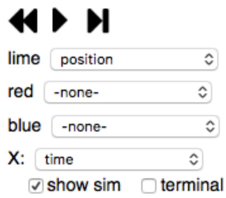
Click the Sim tab:


Make sure that you set the following conditions:




Run the simulation and click the Time Graph tab.

Choose to show 'position' only:




*(6)✐Sketch the graph that you observe.*

*(7)✐What do you notice about the amplitude of the oscillation as time increases?*

This is known as 'light damping'

*Heavy damping (energy removed quickly)*

Return to the Sim tab and increase the damping to 10.0:

Now run the simulation and click on the Time Graph tab.

*(8)✐Sketch the graph that you observe.*

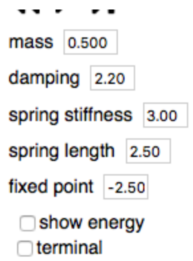
*(9)✐Do you notice any oscillations?*

*(10)✐How would you describe the graph?*

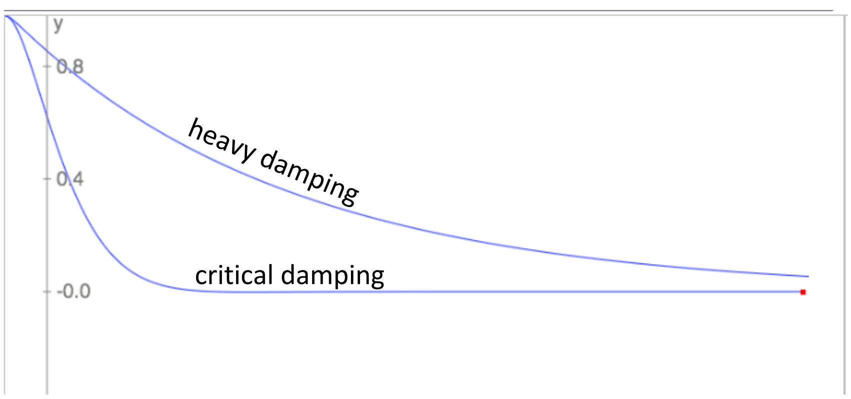
This is known as 'heavy damping'.

*Critical damping*

Ruturn to the Sim tab and change the damping to 2.20:




Run the simulation and click on the Time Graph tab. You should see a graph that looks like the following:




This should be similar to what you observed for heavy damping, except the system returns to the equilibrium position more quickly. In fact, the definition of critical damping is that the system returns to the equilibrium position in the shortest posible time without oscillating.