6.1.3 On the road

Objects undergoing uniform circular motion have a resultant force acting towards the centre of the circle. This is called the centripetal force. If a question involves an object moving in a circle (or part of a circle), the resultant force acting on the object will provide the centripetal force.

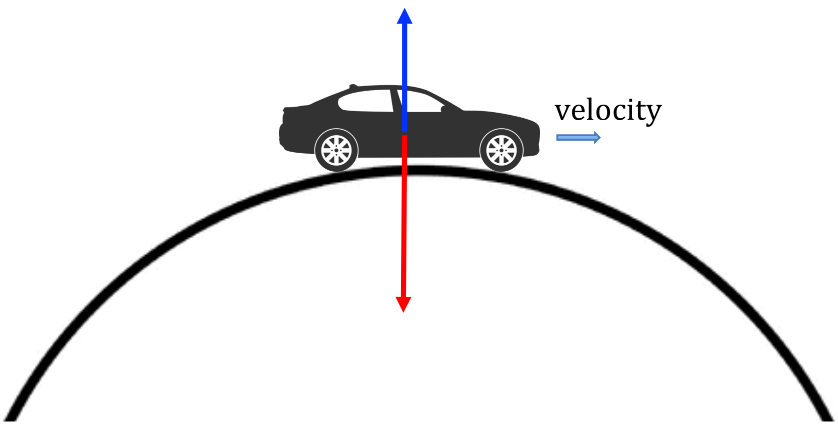
Three common problems involve hills, roundabouts and banked tracks.

The key to any circular motion problem is to identify the forces acting on the object and then work out the resultant force.

Hills and hump backed bridges

Consider a vehicle moving along a circular path over a hill.

(1)✐ Label the forces acting on the vehicle.



The **resultant** of the two forces acts as the **centripetal force** keeping the vehicle moving in a circle.

(2)✐ Write down an expression for the resultant force acting on the vehicle.

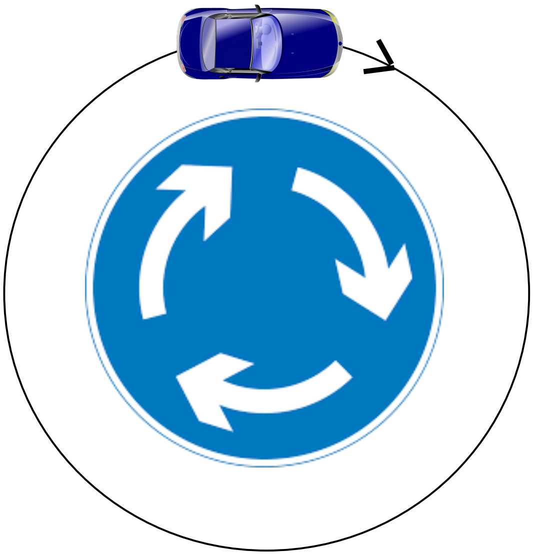
(3)✐ Using your prior knowledge (or look it up!), write down the formula for centripetal force.

(4)✐ Now equate this with the expression for resultant force.

(5)✐ Now write an expression for the speed (v_0) at which the car would just lift off the road?

Roundabouts

(6)✐ What provides the centripetal force for cars moving in a circle around a roundabout? (Be specific)



The limiting force of friction is given by $\mathbf{F_0 = \mu mg}$, where $\boldsymbol{\mu}$ is the coefficient of friction, $\boldsymbol{m}$ is the mass of the car, and $\boldsymbol{g}$ is the gravitational field strength.

(7)✐ Show that the maximum speed v_{max} for no slipping is given by the expression:

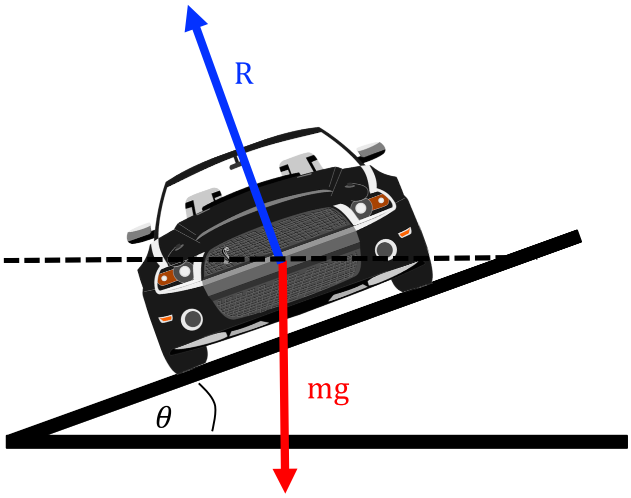
$$\boldsymbol{v_{max} = \sqrt{\mu g r}}$$

(8)✐ What 2 things could increase the speed at which the vehicle can go around the roundabout?

Banked tracks

A racetrack is often angled (banked) at the corners.

(9)✐ Why do you think that this is done?



The two forces acting on the vehicle are the weight (***mg***) and the normal reaction (***R***) from the inclined track. The track is inclined at an angle θ.

Given that the vehicle is moving in a horizontal circle, we know that the resultant force (centripetal force) must be a horizontal force, acting towards the centre of the circle (to left).

R is the only force which can provide a horizontal force component.

(10)✐ Write down an expression for the horizontal component of R.

The horizontal component of R is therefore the centripetal force.

(11)✐ Now equate the horizontal component of R with the equation for centripetal force.

Because the car is remaining at the same height (not moving vertically), the upward component of R should balance the downward component (weight).

(12)✐ Write an expression for the vertical component of R and equate it to the weight.

Now the challenging step!

(13)✐ Using your equations from above show that the car can corner without requiring any friction, when the speed v is given by the following:

$$v = \sqrt{gr\tan\theta}$$