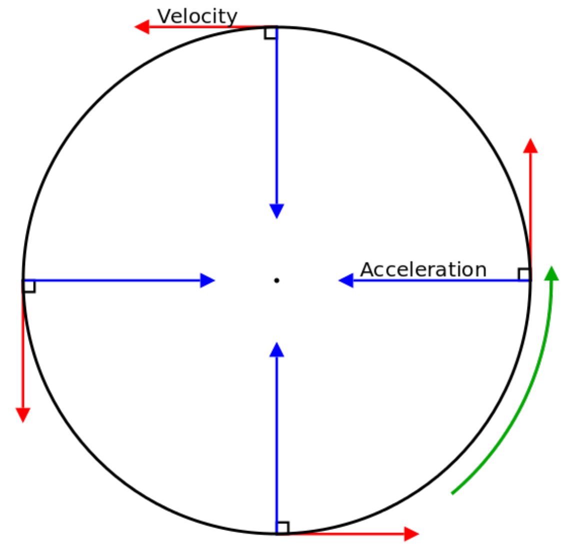
6.1.2 Centripetal Acceleration

"Objects will move at a constant speed in a straight line, unless acted upon by an unbalanced force." (Newton's 1st law).

You know the "unbalanced force" as the **resultant force.**

If a resultant force acts in the same direction as the motion, the object will accelerate. If the resultant force acts in the opposite direction, the object will decelerate. If the resultant force acts at right angles, the object will change direction, **without changing speed**.



(1)✐ In the case of uniform circular motion, in which direction does the resultant force always act?

(2)✐ What is the special name given to this resultant force?

We have now established that the speed of the object undergoing uniform circular motion will remain constant.

(3)✐ Using your knowledge of how acceleration is calculated, explain how the object can be accelerating whilst moving at a constant speed.

Since the acceleration always acts in the same direction as the resultant force an object undergoing circular motion will always be accelerating towards the centre of the circle.

(4)✐ What is the special name given to this acceleration?

The following expression is how this acceleration is calculated: $a = \frac{v^2}{r}$, where v is the speed and r is the radius of the circle.

(5)✐ Using your expression from the previous core task, that links linear speed and angular speed, rewrite the formula to show how acceleration may be calculated using angular speed

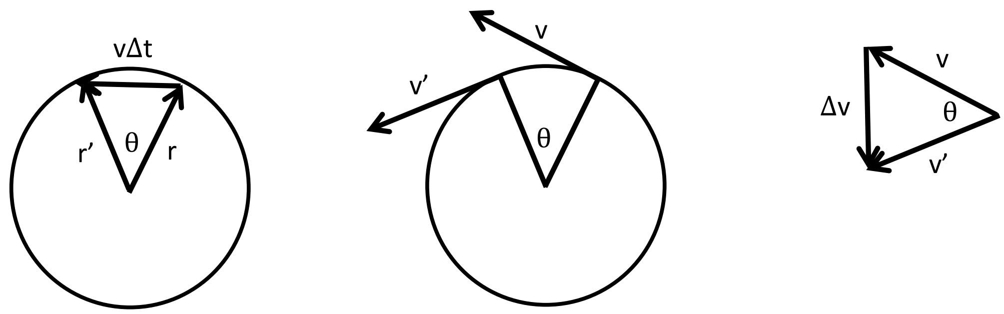
(6)✐ *Using the common expression of Newton's 2nd law (F=ma), write 2 expressions for the centripetal force,*

1. *In terms of v*

2. *In terms of ω*

Derivation of the formula $a = \frac{v^2}{r}$

(Note: this is not required for the specification, but helps your understanding)



(a) (b) (c)

$$acceleration = \frac{change\ of\ velocity}{change\ of\ time} = \frac{\Delta v}{\Delta t}$$

The secret to proving the relationship is recognising that the angle ϴ that is moved through in a given time *t*, is the same for both the displacement vectors *r* and *r'*, and the velocity vectors *v* and *v'*. See https://phet.colorado.edu/en/simulation/legacy/ladybug-motion-2d

$$\theta = \frac{\Delta r}{r} = \frac{\Delta v}{v}$$

But for small angles: $\Delta r = v\Delta t$

Therefore:

$$\theta = \frac{v\Delta t}{r} = \frac{\Delta v}{v}$$

(7)✐ *Do the final step of rearranging, to show that you get the desired formula for acceleration.*