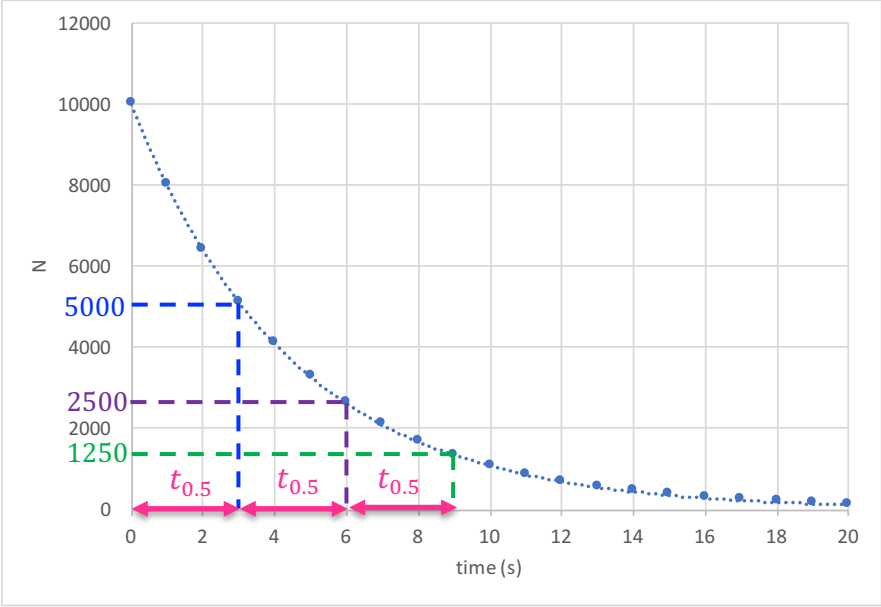
4.5.1 Half-life

videos

Certain nuclei are unstable and emit alpha (α) or beta (β) particles and gamma (γ) radiation. We say that they are radioactive. This is a completely random process. If we have a collection of radioactive nuclei, we can't say when a particular nucleus will decay. However, we do know that approximately half of the nuclei will decay in a certain time, called the 'half-life'.

"Half-life is the time it takes for half of the original radioactive nuclei to decay."

Consider the following graph. It shows how the number of undecayed radioactive nuclei N varies with time. As nuclei decay, N decreases in a characteristic way (called 'exponential decay').



We can see that to start there are 10000 nuclei. After a period of time, called the half-life ($t_{0.5}$), the number has dropped to 5000.
(1) What is the value of the half-life, in this case?

A characteristic of radioactive decay is that whatever number of radioactive nuclei you start with, it always takes the same amount of time (the half-life) for half of them to decay. For example, if we start with 5000 nuclei, there will be approximately 2500 left undecayed after a time interval of one half-life. You can see this from the graph. If we start with 2500, there will be approximately 1250 left undecayed after a time interval of one half-life.

(2) Starting with 8000 nuclei, approximately how many would remain undecayed after i) one half-life?, ii) two half-lives?

If we start with a number N_0 of radioactive nuclei, the number left undecayed after one half-life is given by:

$$N_{left} = 0.5 \times N_0$$

The number left after 2 half-lives is given by:

$$N_{left} = 0.5 \times 0.5 \times N_0$$
$$= 0.5^2 \times N_0$$

The number left after 3 half-lives is given by:

$$N_{left} = 0.5 \times 0.5 \times 0.5 \times N_0$$
$$= 0.5^3 \times N_0$$

In general, the number left after n half-lives is given by:

$$N_{left} = 0.5^n \times N_0$$

Worked example

Question:

A radioactive isotope has a half-life of 4 hours. If there are 200000 nuclei to start, how many would remain undecayed after 2 days?

Answer:

2 days is 48hours, or 12 half-lives (=48/4).

$$N_{left} = 0.5^n \times N_0$$
$$= 0.5^{12} \times 200000$$
$$= 49$$

The same method can be used even if n is not a whole number of half-lives.

(3) A radioactive isotope has a half-life of 30 hours. If there are 500000 nuclei to start, how many would remain undecayed after 2 days?

Mass of sample

Sometimes questions are asked where the mass of a radioactive isotope is given rather than the number of nuclei. Half-life can also be expressed in terms of the mass:

"Half-life is the time it takes for the mass of a radioactive isotope to decrease by half."

Worked example

Question:

A radioactive isotope has a half-life of 20 seconds. If there is a mass of 12g of the isotope to start, how much would remain undecayed after 3 mins?

Answer:
3 mins is 180 seconds, or 9 half-lives (=180/20).

$$mass_{left} = 0.5^9 \times mass_{start}$$
$$= 0.5^9 \times 12$$
$$= 0.023g$$

(4) A radioactive isotope has a half-life of 35 seconds. If there is 200g of the material to start, how much would remain undecayed after 2mins?

Activity

When a radioactive nucleus decays, it gives off radiation. The more nuclei that you have, the more radiation that is given off. Activity is the number of decays that occur in a sample per second. Activity is measured in becquerels (Bq).

As a sample decays, there become fewer radioactive nuclei, and so the activity of a sample decreases in an identical way to the number of undecayed nuclei.

Worked example
Question:
A radioactive isotope has a half-life of 19 seconds. To start with the sample has an activity of 400Bq. What is the activity of the sample after 1min?

Answer:
1 min is 60 seconds, or 3.2 half-lives (=60/19).

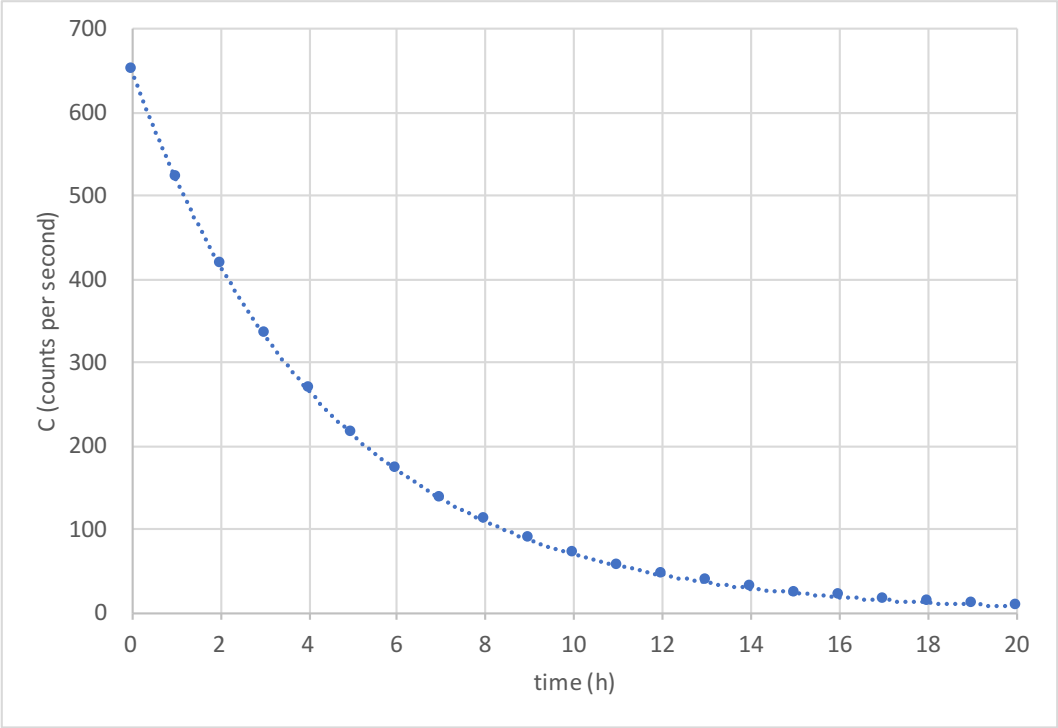
$$activity\ after\ 60\ seconds = 0.5^{3.2} \times activity_{start}$$
$$= 0.5^{3.2} \times 400$$
$$= 44Bq$$

(5) A radioactive isotope has a half-life of 45days. To start with, the sample has an activity of 1200Bq. What is the activity of the sample after 30 days?

Count rate

A Geiger-Muller (GM) tube, connected to a counter can be used to monitor the radiation given out by a sample (see section 4.4.1). It will detect only a fraction of the total amount of radiation given out. Every time a radiation (alpha, beta, gamma) is detected, it is registered as a 'count'. The number of counts per second is known as the 'count rate' (C).

Consider the graph, below. This shows how the count rate of a sample of radioactive isotope changes with time.



(6) Express half-life in terms of the count rate.

(7) What is the half-life of this radioactive isotope?

(8) A larger sample of the same radioactive isotope is found to give a count rate of 55000Bq. What is the count rate after 7hours?