

3.5.1 Single slit diffraction

Waves passing through a single slit will also diffract and produce an interference pattern. The reason for this is to do with the finite width of the slit. We will consider this later.

Take a look at the following simulation:

http://www.walter-fendt.de/html5/phen/singleslit_en.htm

Change the wavelength of the incident light.

(1) *What happens to the spacing of the bright fringes on the screen when the wavelength is increased?*

The spacing increases

Now change the slit width.

(2) *What happens to the spacing of bright fringes on the screen when the slit width is increased?*

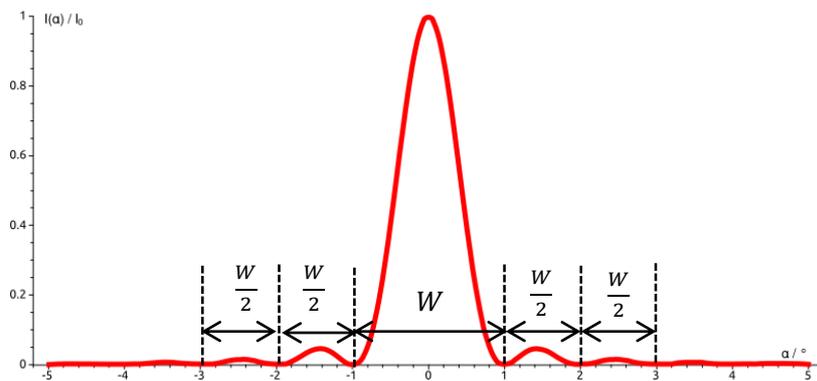
The spacing decreases

It is found that the width of the central bright fringe is given by the expression:

$$W = \frac{2\lambda D}{a}$$

where λ =wavelength, D =slit to screen distance, a =slit width

If we look at the intensity pattern for the light on the screen we get something like the following:



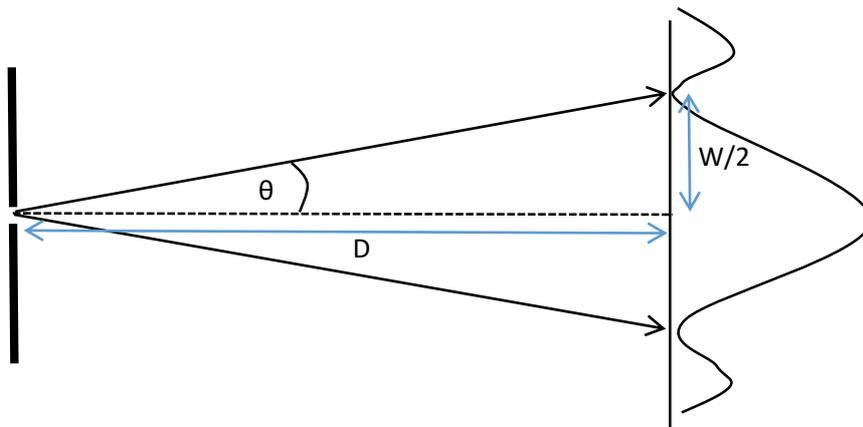
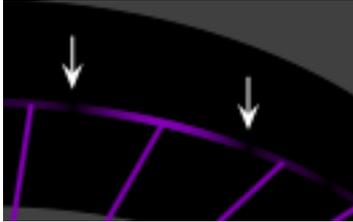
We can see that the central bright fringe has a width W . Subsequent bright fringes have half the width of the central fringe. Note, too that the intensity falls rapidly from central fringe to subsequent fringes.

(3) *Find the width of the central bright fringe, when $a=1 \times 10^{-5} \text{ m}$, $D=3 \text{ m}$, and the $\lambda=450 \text{ nm}$.*

$$W = \frac{2 \times 450 \times 10^{-9} \times 3}{1 \times 10^{-5}} = 0.27 \text{ m} = 27 \text{ cm}$$

Let us return to the simulation above.

You will see that instead of finding W , an angle can be found by moving the sliders until the white arrows point to the dark fringes either side of the central bright fringe.



We can see how the angle is related to W .

$$\frac{W/2}{D} = \tan \theta$$

Therefore:

$$W = 2D \tan \theta = \frac{2\lambda D}{a}$$

And finally:

$$\tan \theta = \frac{\lambda}{a}$$

(4) *In the simulation, select $\lambda=550\text{nm}$, $a=1500\text{nm}$. Move the angle slider until the arrows point to the first dark fringe. Record the angle. Now use the formula above to calculate the angle. Show your working. You should get the same answer!*

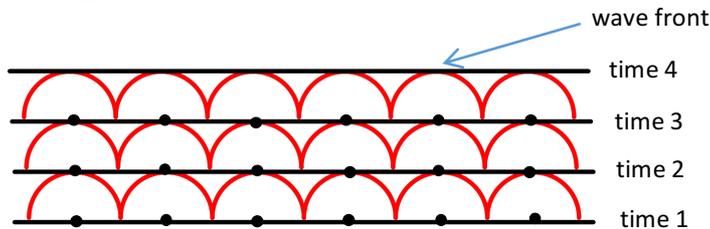
The measured angle is around 20° .

$$\tan \theta = \frac{550 \times 10^{-9}}{1500 \times 10^{-9}} = 0.366$$

$$\tan^{-1}(0.366) = 20.1^\circ$$

Huygens' theory of wave propagation

To explain how wave fronts move forward, the Dutch scientist Christiaan Huygens suggested that you should consider each point on the wave front as a source of secondary wavelets that spread out in all directions. It is the combination of these small wavelets that produce an advancing wave front.

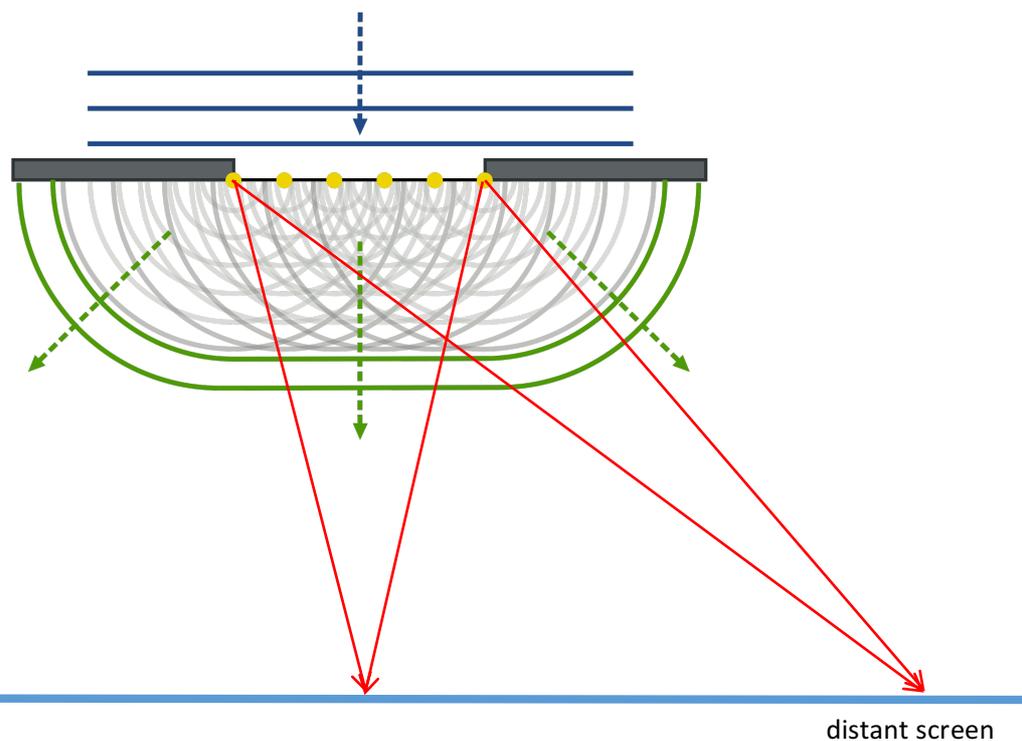


Have a look at the following simulation:

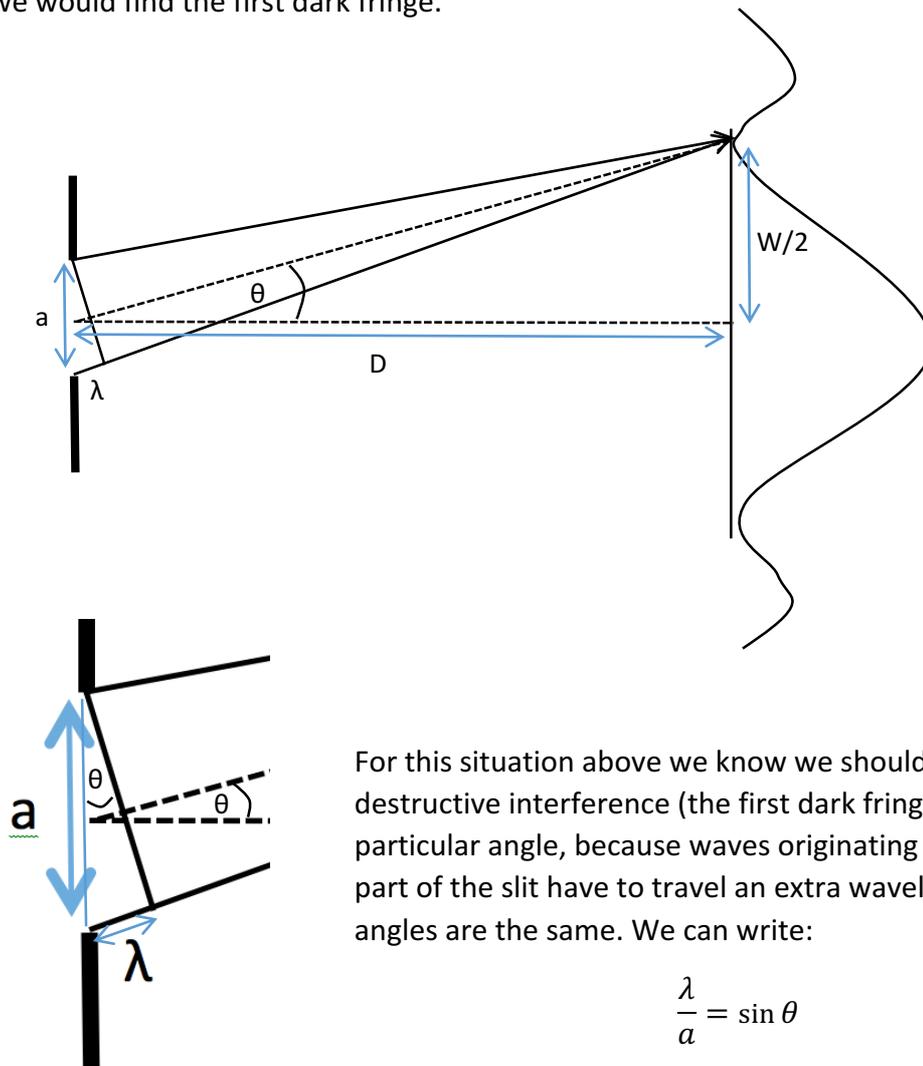
http://www.walter-fendt.de/html5/phen/refractionhuygens_en.htm

In this simulation you will see a wave front being refracted and reflected at a boundary between two media. If you click Next step you will see the Huygens explanation for these processes.

Now consider waves passing through a gap. Waves diffract around the edges. Straight ahead all the waves from all points will arrive at a distant screen roughly in phase. However at an angle, waves will arrive at the screen with a phase difference because the path length will be different for waves originating from different points.



Consider the diagram above. At a certain angle waves from the far left of the slit will have to travel an extra wavelength more than waves from the far right of the slit. This means they would be in phase. However, waves from the middle will have to travel half a wavelength further and so will be in antiphase. If we add up all the waves from points all the way along the gap, we will add waves for one whole cycle, i.e. with phase differences 0-360°. When we do this we end up with them all cancelling each other out – destructive interference. At this point we would find the first dark fringe.



For this situation above we know we should get destructive interference (the first dark fringe) at this particular angle, because waves originating from the lower part of the slit have to travel an extra wavelength λ . The angles are the same. We can write:

$$\frac{\lambda}{a} = \sin \theta$$

and:

$$\frac{W/2}{D} = \tan \theta$$

The small angle approximation ($\sin \theta = \tan \theta$) can be used here because D is very large compared to a . Therefore:

$$\frac{\lambda}{a} = \frac{W/2}{D}$$

Hence:

$$W = \frac{2\lambda D}{a}$$

as shown at the top.

(5) *If the 1st dark fringe is found at an angle θ , at what angle would you find the next dark fringe?*

The width of the central bright fringe is W , so the distance to the first dark fringe is $\frac{W}{2}$.

Subsequent bright fringes have a width of $\frac{W}{2}$. This means that the angle to the second dark fringe is just 2θ .

