1.2.1 Limitations of physical measurements

Whenever we measure something using a measuring instrument there are errors introduced that arise from the instrument used and the procedure followed. This will give rise to an uncertainty in the measured value.

videos

(1)✎Make a list of how errors could arise in taking a measurement of the period of a pendulum (i.e. the time it takes for the pendulum to complete one swing – there and back).

An uncertainty is a measure of how close we can be sure that the measured value is to the true value.

An example:

If we measure the length of a piece of paper using a standard ruler, we might find that it measures *29.7cm*. Because our ruler has a smallest division of *1mm* or *0.1cm*, we can't claim a greater precision than *1mm*. So, we express this uncertainty as:

length of paper = *29.7±0.1cm*

We are saying that we know that the true value is somewhere in the range from *29.6* to *29.8cm*

Random and systematic errors

Some errors are random errors. These are caused by random effects that can cause the instrument reading to be higher or lower than the true value. The effects of random error can be reduced by repeating and averaging.

Some errors are systematic errors. These can be caused by a badly calibrated instrument or by zero error (where the instrument doesn't read zero when it should). Systematic errors result in readings which are consistently too high or too low. Systematic errors cannot be removed by repeating and averaging.

(2)✎What can you do to remove a systematic error?

Quantifying uncertainties

When scientists quote a value for something they should include a measure of the uncertainty in that value.

Measuring something once

Usually when we measure something once, the uncertainty is *±smallest scale division* on the measuring instrument. For example: measuring the length of something with a metre rule will give an uncertainty of *±1mm*. In some cases, this may not be justified. For example, when timing the period of a pendulum using a stopwatch which reads to *0.01s*, we can't claim that the uncertainty in the time we measure is *±0.01s* because the reaction time of the person timing is around *0.2s*. We need to be realistic. A more realistic uncertainty, in this case, will be *±0.2s*.

Repeat readings

If we measure the same thing more than once, the spread of readings will give us an idea of the uncertainty, and a better idea of the true value. Our best estimate of the true value will be the mean of the readings. The uncertainty is taken to be $\pm\left(\frac{largest\ reading - smallest\ reading}{2}\right)$.

(3)✐Find the best estimate of the true value and the uncertainty for the following readings of voltage: 3.45V, 3.39V, 3.30V, 3.56V.

voltage = ____________ *±*__________ *V*

Combining uncertainties

Adding and subtracting

When we add or subtract two quantities, we just add together the two uncertainties.

For example: the lengths of two objects are measured and a combined length is calculated.

length of A=*3.5±0.1cm*, length of B=*5.2±0.4cm*
the combined length of A+B = *8.7±0.5cm*
the combined length of B-A = *1.7±0.5cm*

(4)✐the thickness of a coin is measured to be 1.20±0.15mm. What is the thickness of 10 coins stacked on top of each other?

Multiplying and dividing

When we multiply or divide quantities we add together the percentage uncertainties of the quantities to work out an overall percentage uncertainty.

A percentage uncertainty is the uncertainty of the quantity expressed as a percentage. For example: The mass of a substance = 10.0±0.1g. This gives a percentage uncertainty of $\frac{0.1}{10.0} \times 100 = 1\%$.

An example:
We wish to calculate the area of a piece of A4 paper. The dimensions are length = *29.7±0.1cm*, width = *21.0±0.1cm*.

area = length x width = *29.7 x 21.0 = 623.7cm²*

$$\text{percentage uncertainty in area} \\ = \text{percentage uncertainty in length} \\ + \text{percentage uncertainty in width}$$

$$\therefore \text{percentage uncertainty in area} = \frac{0.1}{29.7} \times 100 + \frac{0.1}{21.0} \times 100 \\ = 0.34 + 0.48 \\ = 0.82\%$$

We can therefore write:

area = *623.7 cm² ± 0.82%*

or calculate the absolute uncertainty (=*0.82% of 623.7*) = *5.1 cm²*

Therefore:
Area = *623.7 ± 5.1 cm²*

(5)✐Work out a value for the speed of light by dividing distance by time, and work out the uncertainty in the speed.
distance = $3000.0 \pm 0.1m$, *time* =$1.01 \times 10^{-5} \pm 0.01 \times 10^{-5} s$.

Powers

If we wish to work out the uncertainty in a power we just work out the percentage uncertainty in the quantity and multiply by the power.

An example:
We wish to work out the volume of a sphere with radius $r=6.5\pm0.2m$.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 6.5^3 = 1150\ m^3$$

The percentage uncertainty in r is $\frac{0.2}{6.5} \times 100 = 3.1\%$

The percentage uncertainty in $r^3 = 3 \times 3.1 = 9.3\%$. This is also the percentage uncertainty in the volume as there are no other measured values in the formula.

Therefore, we write:
volume $= 1150 \pm 9.3\%$
$= \underline{1150 \pm 110m^3}$

(6)✎Find the density of a cube of material with side length = 3.4±0.1cm and a mass of 0.4kg, and the uncertainty in density. We will assume that the uncertainty in mass is negligible.