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7.4 Orbits

When a smaller body is in motion around a larger body, we say that it orbits the larger body. The orbiting smaller body is called a satellite. The planets are natural satellites of the Sun, and the Moon is a natural satellite of the Earth. Bodies in stable orbits follow elliptical paths.

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Let us consider one special ellipse – a circle! From section 6.1, we know that a condition of circular motion is that the resultant force acting on a body is a force which acts towards the centre of the circle. This is called the centripetal force (F_c):

$$F_c = \frac{mv^2}{r}$$

where m=mass of body, v=speed, r=radius of circle.

In the case of orbits, the centripetal force comes from the gravitational attraction between the two bodies:

$$F_g = \frac{GMm}{r^2}$$

where G=gravitational constant, M=mass of larger body, m=mass of smaller body, r=distance between the centres of mass of M and m.

(1) By equating $F_c = F_g$ and simplifying, show that the orbital speed $v = \sqrt{\frac{GM}{r}}$.

(2) How is the speed related to the mass of the orbiting body?

The speed of orbit is related to the period T (time for one orbit) in the following way:

$$v = \frac{2\pi r}{T}$$

(3) Substitute this into the equation for speed, above, and simplify to show that $T^2 \propto r^3$.

(4) The orbital radius of Mars is approximately 1.5 times that of the Earth. Work out the orbital period of Mars using the relationship above.

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Energy considerations

Orbiting bodies have a combination of gravitational potential and kinetic energy. We saw, in section 7.3 that the gravitational potential energy is given by:

$$E_{gp} = -\frac{GMm}{r}$$

where G=the gravitational constant=$6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$*, and r=distance between the centre of mass of* M *and* m*.*

(5)✏Using the equation for orbital speed, substitute it into the equation $E_k = \frac{1}{2}mv^2$*, to get an expression for the kinetic energy.*

(6)✏How does the kinetic energy compare to the gravitational potential energy for an orbiting body?

When the orbital radius of a body changes, there is a conservation of total energy:

$$E_{total} = E_{gp} + E_k = constant$$

(7)✏What would happen to the speed of an orbiting body if its radius decreases? Explain why.

Escape velocity

If we wish to launch an object into space so that it never returns, then we must give it enough kinetic energy ($E_{k\ launch}$) to exceed the gravitational potential energy "well" that it is in.

$$E_{k\ launch} \geq E_{gp}$$
$$\therefore \frac{1}{2}mv^2 \geq \frac{GMm}{r}$$

(8)✏Simplify to get an expression for the escape velocity v_e *(the velocity where there is just enough kinetic energy to escape).*

(9)✏Is the escape velocity dependent on the mass of the object?

(10)✐Work out the escape velocity from the surface of the Earth. $r_E = 6.4 \times 10^6 m, m_E = 6 \times 10^{24} kg$).

Geosynchronous orbits

A geosynchronous orbit is one in which the smaller body orbits with a period equal to the rotation of the larger body. If the body orbits in the same direction as the rotation this will mean that the body will remain over the equator, at a fixed position. Artificial satellites in geosynchronous orbits around the Earth can be useful for communications. For example, satellite dishes can remain pointing in one direction, and don't need to move.

(10)✐Given that the rotation period of the Earth is roughly 24 hours, calculate the radius of a geosynchronous orbit.