6.7 Kinetic theory model of gases

We have already seen how the pressure (P), volume (V) and temperature (T) in gases can be described by the Gas Laws. These laws were discovered in the 17th and 18th centuries, by experimentation. The General Gas Law, is an amalgamation of these laws and states that:

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$$PV = nRT$$

where R=molar gas constant, n=number of moles

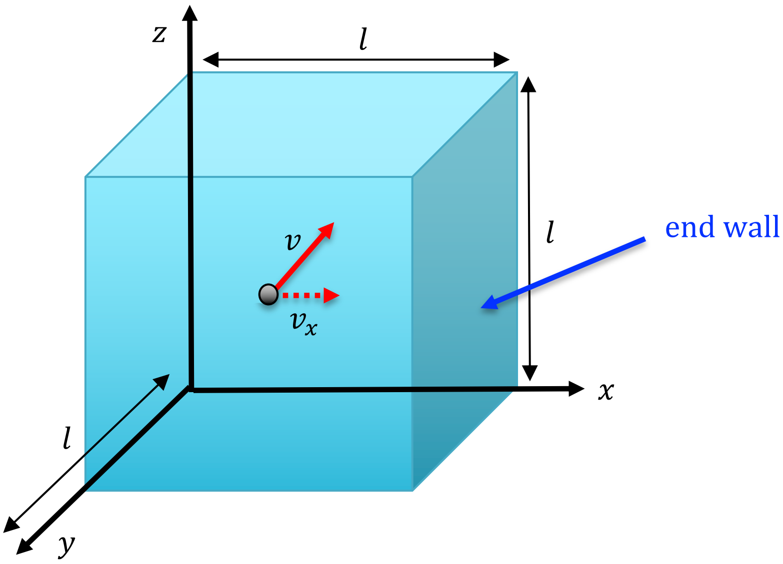
The next step was to explain this relationship using a mathematical model. To describe the behavior of gases mathematically, we must make certain assumptions. These are:

1) *Gases consist of small particles that are in random motion.*
2) *The volume of the gas particles is negligible compared to the volume of the gas itself.*
3) *All collisions are elastic – that is, kinetic energy is conserved.*
4) *There are no interactions between particles, except during collisions.*

(1)✐Do the particles in this model have kinetic energy?

(2)✐Do the particles in this model have potential energy?

Consider a particle moving with velocity v in a square box with side length l.



The component of velocity in the x direction is v_x. It will also have component of velocity in the y and z directions, but we will only consider the motion in the x direction, for the moment.

The time between collisions (Δt) of the particle with one of the end walls (in the x direction) will be:

$$\Delta t = \frac{distance\ travelled}{speed} = \frac{2l}{v_x}$$

The change of momentum (Δp) for the particle bouncing off the wall (with no loss of kinetic energy) will be:

$$\Delta p = -mv_x - mv_x = -2mv_x$$

where m is the mass of the particle.

The average force on the wall can be calculated from the rate of change of momentum (Newton's 2nd law):

$$F = \frac{\Delta p}{\Delta t}$$

Therefore, the magnitude of the force on the end wall is given by:

$$F = \frac{2mv_x}{\left(\frac{2l}{v_x}\right)} = \frac{mv_x^2}{l}$$

The pressure (p_x) on the end wall can be calculated:

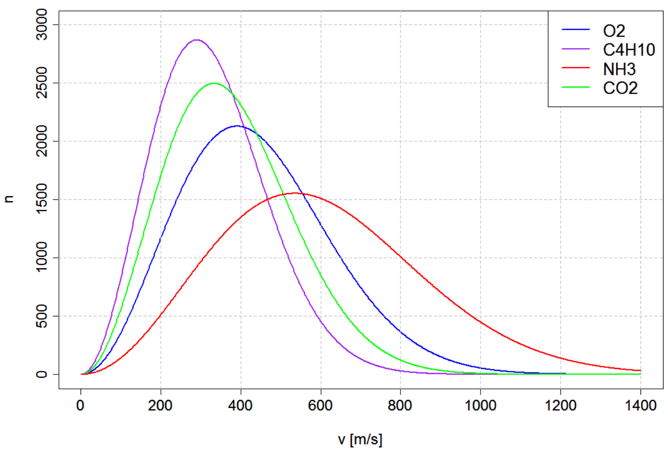
$$p_x = \frac{F}{A} = \frac{mv_x^2}{(l \times l) \times l} = \frac{mv_x^2}{V}$$

where V is the volume of the box

If there are N particles, then the total pressure on the end wall is given by:

$$P = \frac{Nmv_x^2}{V}$$

However, the x component of velocity is not the same for all particles, because they are moving in random directions and there is a distribution of velocities.



The graph, left, shows how the speed of particles in a gas is distributed.

Instead, we will use the mean square value $\overline{v_x^2}$ of all the velocities.

$$P_x = \frac{Nm\overline{v_x^2}}{V}$$

note: $\overline{v_x^2} = \frac{v_{1x}^2+v_{2x}^2+v_{3x}^2\cdots}{N}$, for particles with different x components of velocity.

(3)✐Four particles have the following x components of velocity:
$+123ms^{-1}, -201ms^{-1}, -120ms^{-1}, +133ms^{-1}$. *Work out the mean square velocity in the x direction.*

For a single particle:

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

For many particles, we use the **root mean square** value for the overall, representative magnitude of velocity. This is written v_{rms}. To find the root mean square, you square all the values, take the mean and finally, take the square root.

(4)✐Four particles have the following velocity magnitudes:
$303ms^{-1}, 291ms^{-1}, 288ms^{-1}, 320ms^{-1}$. *Find the root mean square value of velocity.*

As the direction of motion of particles is totally random, the average component of velocity in the *x, y and z* directions will be $\frac{1}{3}$ of v_{rms}. Hence the pressure on any wall is given by the expression:

$$P = \frac{\frac{1}{3}Nmv_{rms}^2}{V} \qquad (1)$$

(5)✐Given that Nm is the total mass of particles in the box, show that the expression, above can be written:

$$P = \frac{1}{3}\rho v_{rms}^2$$

where ρ is the density of the gas

(6)✐By equating the molar gas equation (PV=RT), to equation (1), above, show that:

$$kT = \frac{1}{3}mv_{rms}^2 \qquad (2)$$

where k = the Boltzmann constant = $\frac{R}{N_A}$*, and* N_A *= Avogadro's number*

If we take equation (2), above and multiply both sides by $\frac{3}{2}$ we get the following expression:

$$\frac{3}{2}kT = \frac{1}{2}mv_{rms}^2$$

The expression on the right of the equals sign should look familiar. It is an expression for the mean <u>kinetic energy</u> of the gas particles.

(7)✐How would you describe the relationship between the temperature and mean kinetic energy of the gas particles?

(8)✐Find the root mean square velocity (v_{rms}) of nitrogen gas particles at room temperature. (Note: Nitrogen gas is N_2, take room temperature = 300K)