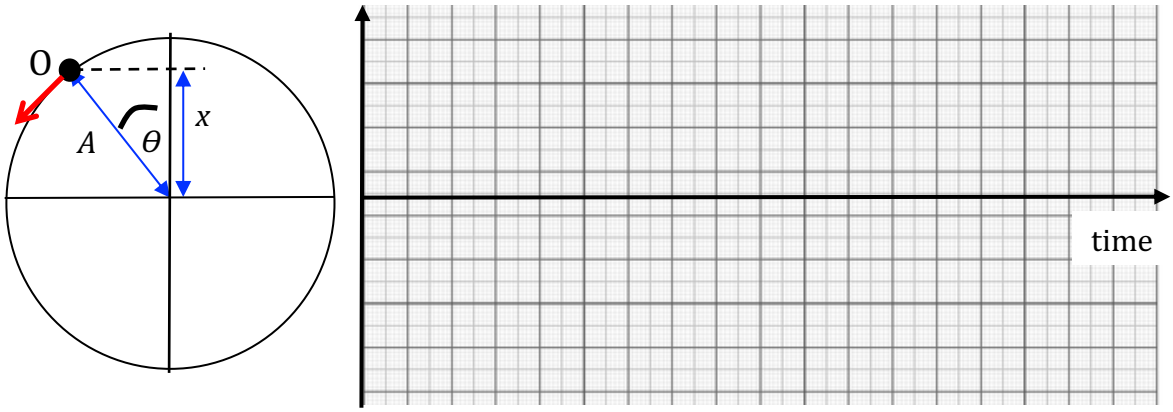
6.2.3 More about sine waves

There is a link between Simple Harmonic Motion (SHM) and Circular Motion. See
https://www.youtube.com/watch?v=0hp6Okk_tww

videos



Imagine the point O is rotating with uniform circular motion, anticlockwise (from the top) at a constant angular speed of 30° per second (or π/6 rad s^{-1}).

(1) Plot the vertical displacement of the point with time on the graph. Choose a time scale such that you show one complete cycle.

(2) What function describes the shape of this graph?

If A is the radius of the circle, then the vertical displacement is given by:

$$x = A\cos\theta$$

where θ is the angle shown in the diagram

However, θ varies with time according to the following:

$$\theta = 2\pi f t,$$

where f is the frequency of the rotation and t is the time

Hence the function describing the displacement-time graph on the right is:

$$x = A\cos(2\pi f t)$$

or:

$$x = A\cos\,\omega t,$$

where $\omega = 2\pi f$ (= angular speed)

The velocity graph is the gradient of the displacement-time graph. Mathematically, this is the differentiation of the displacement function (with respect to t).
Hence:

$$v = -A\omega\,\sin\omega t$$

(3) Rewrite this equation, substituting in for $\omega = 2\pi f$.

(4) Write an expression for the maximum velocity v_{max}. (Hint: What is the largest value the sin function can take?

The acceleration graph is the gradient of the velocity-time graph.
Mathematically, this is the differentiation of the velocity function (with respect to t). Hence:

$$a = -A\omega^2 cos\omega t$$

(5) Rewrite this equation, substituting in for $\omega = 2\pi f$.

(6) Write an expression for the maximum acceleration a_{max}.

We know that the defining feature of SHM is:

$$a = constant \times (-x)$$

(7) From the equations for x and a, above, rewrite this equation substituting in an expression for the constant.