5.6 Emf and internal resistance

In a circuit, charges pick up energy in the power source and distribute it to components around the circuit. Voltage is a measure of the energy that charges carry (or energy per unit charge):

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$$V = \frac{E}{Q}$$

where E = energy (in joules) and Q = charge (in coulombs)

A distinction is made between the energy that the charges pick up and the energy that the charges give up to the circuit components.

The energy per unit charge picked up by charges is called the electromotive force (emf). It is represented by the symbol ε. Electromotive force is measured in volts:

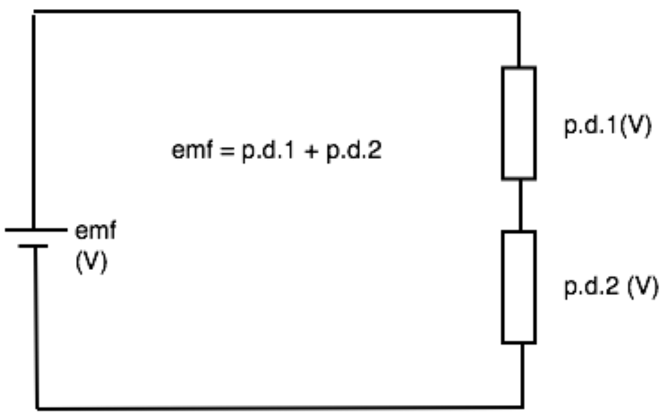
$$\varepsilon = \frac{E}{Q}$$

The energy per unit charge given up by charges passing through a circuit component is called the potential difference ($p.d.$). Potential difference is also measured in volts:

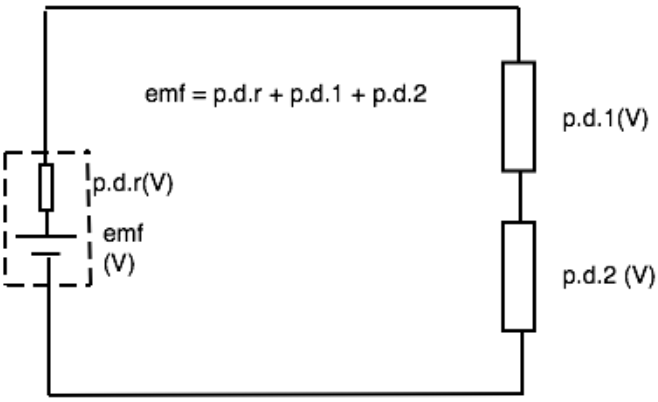
$$p.d. = \frac{E}{Q}$$

In a circuit the emf supplied to charges is equal to the sum of $p.d.$s around a circuit loop:

$$\varepsilon = p.d._1 + p.d._2 + p.d._3 + \cdots$$



Power sources are seldom 'ideal' and some voltage is dropped inside the power source due to <u>internal resistance</u>.



In the diagram above there is some internal resistance (r), so there is a $p.d.$ dropped across this resistance inside the power supply.
From ohm's law ($p.d. = IR$). Therefore:

$$\varepsilon = Ir + IR$$

where R = combined resistance of components in the external circuit (called the 'load'), and r = internal resistance

The power supplied by the cell (remember $P = IV$) is given by:

$$P = I\varepsilon = I^2 r + I^2 R$$

The power delivered to the external circuit is therefore:

$$I^2 R = I\varepsilon - I^2 r$$

This shows that the power delivered to the external circuit is the total power supplied by the power source ($I\varepsilon$) minus the power lost internally ($I^2 r$).

The current flow (I) in the circuit is given by:

$$I = \frac{\varepsilon}{R + r}$$

(1)✏By substituting this for I on the right-hand side of the equation for power delivered to the external circuit, simplify and show that:

$$I^2 R = \frac{\varepsilon^2 R}{(R + r)^2}$$

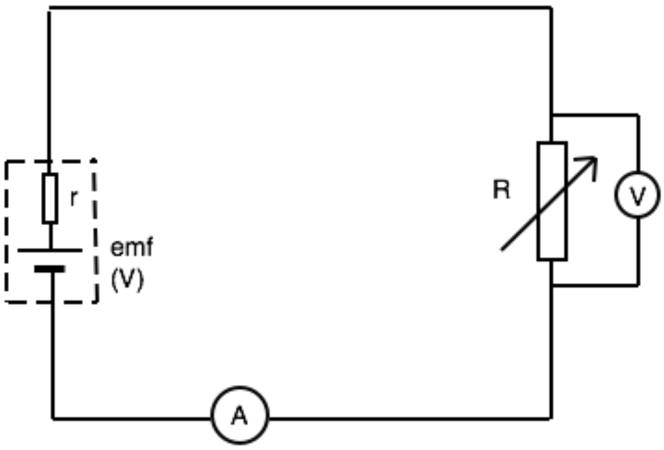
(2)✏What happens to the power delivered to the external circuit when R is very large (compared to r)?

(3)✎What happens to the power delivered to the external circuit when R is very small (compared to r)?

(4)✎At what value of R do you think that the power supplied to the external circuit reaches a maximum?

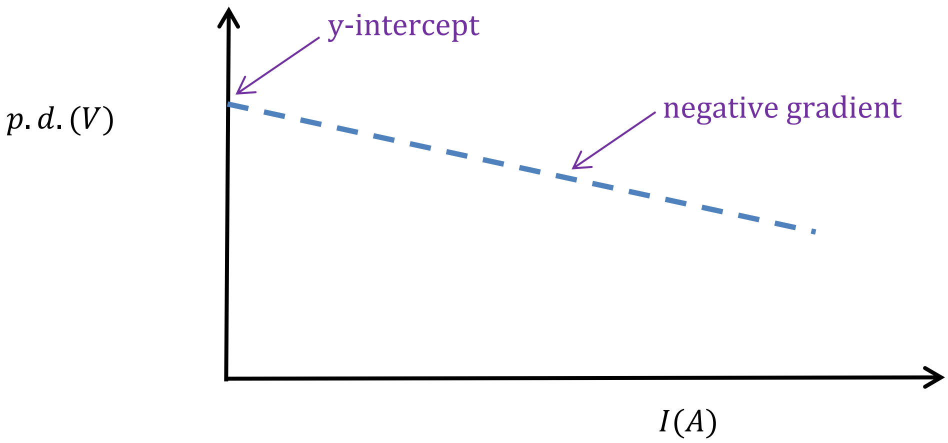
Measuring the internal resistance

The following circuit can be used to measure the emf and the internal resistance of a power supply.



The variable resistor is used to alter the current flowing in the circuit. Current is measured with an ammeter and the $p.d.$ across the variable resistor is measured with a voltmeter.

The voltage is plotted against current.



If we look at the equation for voltages:

$$\varepsilon = Ir + IR$$

Rearranging to get the $p.d.$ across the external resistor:

$$IR = \varepsilon - Ir = (-r)I + \varepsilon$$

This is in the form of $y = mx + c$ for a straight line. (where m is the gradient of the line, and c is the y-intercept).

(5)✐From the graph (above), what does the gradient give you?

(6)✐From the graph (above), what does the y-intercept give you?

Note: the y-intercept occurs when the current is zero. This means that R must be infinitely large. Essentially we are measuring the voltage across the terminals of the power supply when no current is flowing. This is called an 'open circuit' measurement. As the external resistance (R) falls and larger current flows the terminal voltage falls and as a larger $p.d.$ is dropped across the internal resistance of the power supply. This has important consequences for the use of power supplies. For example, you may measure the terminal voltage of a battery to be 1.5V, using a voltmeter. However, once the battery is connected in a circuit and a current is flowing, a $p.d.$ is dropped across the internal resistance of the battery, reducing the terminal voltage. The bigger the current flowing, the smaller the terminal voltage will be.

(7)✐Some batteries are sold as being suitable for high power devices, whereas others are sold as being suitable for low power devices. How do you think their internal resistances will compare?